181301 TRANSFORMS AND PARTIAL DIFFERENTIAL EQUATIONS

OBJECTIVES

The course objective is to develop the skills of the students in the areas of Transforms and Partial Differtial Equations. This will be necessary for their effective studies in a large number of engineering subjects like heat conduction, communication systems, electro-optics and electromagnetic theory. The course will also serve as a prerequisite for post graduate and specialized studies and research.

1. FOURIER SERIES 9 + 3

Dirichlet's conditions – General Fourier series – Odd and even functions – Half range sine series – Half range cosine series – Complex form of Fourier Series – Parseval's identify – Harmonic Analysis.

2. FOURIER TRANSFORMS 9 + 3

Fourier integral theorem (without proof) – Fourier transform pair – Sine and Cosine transforms – Properties – Transforms of simple functions – Convolution theorem – Parseval's identity.

3. PARTIAL DIFFERENTIAL EQUATIONS 9 +3

Formation of partial differential equations – Lagrange's linear equation – Solutions of standard types of first order partial differential equations - Linear partial differential equations of second and higher order with constant coefficients.

4. APPLICATIONS OF PARTIAL DIFFERENTIAL EQUATIONS 9 + 3

Solutions of one dimensional wave equation – One dimensional equation of heat conduction – Steady state solution of two-dimensional equation of heat conduction (Insulated edges excluded) – Fourier series solutions in cartesian coordinates.

5. Z -TRANSFORMS AND DIFFERENCE EQUATIONS 9 + 3

Z-transforms - Elementary properties – Inverse Z-transform – Convolution theorem -Formation of difference equations – Solution of difference equations using Z-transform.

Lectures : 45 Tutorials : 15 Total : 60

TEXT BOOKS

1. Grewal, B.S, "Higher Engineering Mathematic", 40th Edition, Khanna publishers, Delhi, (2007)

REFERENCES

1. Bali.N.P and Manish Goyal, "A Textbook of Engineering Mathematic", 7th Edition, Laxmi Publications(P) Ltd. (2007)
2. Ramana.B.V., "Higher Engineering Mathematics", Tata Mc-GrawHill Publishing Company limited, New Delhi (2007).
3. Glyn James, "Advanced Modern Engineering Mathematics", 3rd Edition, Pearson Education (2007).
4. Erwin Kreyszig, "Advanced Engineering Mathematics", 8th edition, Wiley India (2007).

MA2211 TRANSFORMS AND PARTIAL DIFFERENTIAL EQUATIONS

TWO MARKS (Q&A)

UNIT-1

[Fourier series]

1. Define periodic function?

A function f(x) is said to be have a period T if for all x ,f(x+T)=f(x),where the T is a positive constant. The least value of T>0 is called the period of f(x).

2. Define Continuous function?

A Continuous function at x=a is denoted $\underset{x\to a}{lt} f(x)=f(a)$,i.e., $\underset{x\to a}{lt} f(x)$ exists.f(x) is said to be Continuous in an interval (a,b)if it is Continuous at every point of the interval.

3. Define Discontinuous function?

A function f(x) is said to be discontinuous at at point if it is not Continuous at that point.

4. Define Fourier series?

If f(x) periodic function and satisfies Dirichlet condition,then it can be represented by an infinite series called Fourier series as

$$f(x)=\frac{a_0}{2}+\sum_{n=1}^{\infty}(a_n\cos nx+b_n\sin nx)$$

5. Define Even functions?

A function f(x) is said to be even if f(-x)=f(x).

6. Define Odd functions?

A function f(x) is said to be odd if f(-x)=-f(x).

7. Pick out the even function : x^2,sinx?

x^2 is an even function ,sinx is an odd function.

8. Write the formula for Fourier constant for f(x) in the interval $(-\pi,\pi)$.

$$a_0=\frac{1}{\pi}\int_{-\pi}^{\pi}f(x)dx\,,a_n=\frac{1}{\pi}\int_{-\pi}^{\pi}f(x)\cos nxdx\,,b_n=\frac{1}{\pi}\int_{-\pi}^{\pi}f(x)\sin nxdx$$

9. Find the Fourier constant a_n when odd function f(x) is expanded in $(-\pi,\pi)$?

$a_n=0$.

10. Find the Fourier constant b_n in the expansion of x^2 in $(-\pi,\pi)$?

Since f(x)= $x^2$is an even function the value of b_n=0

11. What is the sum of the Fourier series at point x= x_0 the eunction f(x) has a finite discontinuity?

f(x)=$\dfrac{f(x+x_0)+f(x-x_0)}{2}$.

12. write Parseval's theorem on Fourier constants?

If the Fourier series corresponding to f(x) converges uniformly to f(x) in $(-l,l)$ then $\frac{1}{l}\int_{-l}^{l}[f(x)]^2dx=\frac{a_0^2}{2}+\sum_{n=1}^{\infty}(a_n^2+b_n^2)$

13. Define Root mean square value of a function?

The root mean square value of f(x) over the interval (a,b)is

$$\text{R.M.S}=\sqrt{\frac{\int_a^b [f(x)]^2\,dx}{b-a}}$$

14. Find the constant a_0 of the Fourier series for the function f(x)=k,0<x<2π.

$$a_0=\frac{1}{\pi}\int_0^{2\pi} f(x)dx=\frac{1}{\pi}\int_0^{2\pi} kdx=\frac{k}{\pi}[x]_0^{2\pi}=\frac{2\pi k}{\pi}=2k.$$

15. Write the Fourier series in complex form for f(x) in the interval c to c+2π?

$$f(x)=\sum_{n=-\infty}^{\infty} C_n e^{inx} \quad \text{where } C_n=\frac{1}{2\pi}\int_c^{c+2\pi} f(x)e^{-inx}dx.$$

16. Write the Fourier series in complex form for f(x) in the interval c to c+2l?

$$f(x)=\sum_{n=-\infty}^{\infty} C_n e^{\frac{in\pi x}{l}} \quad \text{where } C_n=\frac{1}{2\pi}\int_c^{c+2\pi} f(x)e^{\frac{-in\pi x}{l}}dx.$$

17. Write the formula for Fourier constant for f(x) in the interval (c,c+2l)?

$$a_0=\frac{1}{l}\int_c^{c+2l} f(x)dx,\ a_n=\frac{1}{l}\int_c^{c+2l} f(x)\cos\frac{n\pi x}{l}dx,\ b_n=\frac{1}{l}\int_c^{c+2l} f(x)\sin\frac{n\pi x}{l}dx$$

18. Find the Fourier constant b_n for xsinx in $(-\pi,\pi)$?

$b_n=\frac{1}{\pi}\int_{-\pi}^{\pi} f(x)\sin nx dx=\frac{1}{\pi}\int_{-\pi}^{\pi} x\sin x\sin nx dx=0$[$x\sin x\sin nx$ is an odd function].

19. Write the formula for Euler's constant of a Fourier series in 0<x<2π?

$$a_0=\frac{1}{\pi}\int_0^{2\pi} f(x)dx,\ a_n=\frac{1}{\pi}\int_0^{2\pi} f(x)\cos nx dx,\ b_n=\frac{1}{\pi}\int_0^{2\pi} f(x)\sin nx dx$$

20.Find the Fourier series corresponding to f(x)= $x-x^3$ in $(-\pi,\pi)$?

Given f(x)= $x-x^3$,

f(-x)= $-x+x^3$ =-($x-x^3$)= -f(x).

f(-x)= -f(x).

f(x) is an odd function in $(-\pi,\pi)$.**Hence** a_0 =0.

UNIT-2
[Fourier Transforms]

1. Define integral transforms?

The integral transforms of a function f(x)is denoted by L[f(x)]=

$\int_a^b f(x)k(s,x)dx$,s is parameter,f(x) is inverse transform of L[f(x)].

i.e, L[f(x)]= $\int_0^\infty f(x)e^{-sx}dx=\int_0^\infty f(t)e^{-st}dt$

2. Define Fourier integral theorem?

If f(x)is a given function ($-l, l$) and satisfies Dirichlet's condition ,then

f(x)= $\frac{1}{\pi}\int_0^{\infty}\int_{-\infty}^{\infty} f(t)\cos\lambda(t-x)dxd\lambda$

3. Formula for Fourier sine integral?

f(x) **=** $\frac{2}{\pi}\int_0^{\infty}\sin\lambda x\int_0^{\infty} f(t)\sin\lambda t dtd\lambda$

4. **Formula for Fourier Cosine integral?**

f(x) **=** $\frac{2}{\pi}\int_0^{\infty}\cos\lambda x\int_0^{\infty} f(t)\cos\lambda t dtd\lambda$

5. Formula for complex form of Fourier integral?

f(x) **=** $\frac{1}{2\pi}\int_{-\infty}^{\infty}e^{-i\lambda x}\int_{-\infty}^{\infty} f(t)e^{-i\lambda t}dtd\lambda$

6. Define convolution of two function?

If f(x) and g(x) are any two function ($-\infty,\infty$) then the convolution of two function is

f*g= $\frac{1}{\sqrt{2\pi}}\int_{-\infty}^{\infty} f(t)g(x-t)dt$

7. Define parseval's identity?

If f(x) are any given function ($-\infty,\infty$) that it satisfy the identity,

$$\int_{-\infty}^{\infty}|f(x)|^2 dx = \int_{-\infty}^{\infty}|F(s)|^2 ds$$

8. Define finite Fourier Transforms?

If f(x) are any given function ($0,l$) then the finite Fourier sine and cosine Transforms of f(x) in 0<x<l is

$F_s[f(x)] = \int_0^{l} f(x)\sin\frac{n\pi x}{l}dx$

$F_c[f(x)] = \int_0^{l} f(x)\cos\frac{n\pi x}{l}dx$ where 'n' is an integer.

9. Define infinite Fourier Transforms write inverse formula is?

The infinite Fourier Transforms of a function f(x) is

F[f(x)]= $\frac{1}{\sqrt{2\pi}}\int_{-\infty}^{\infty} f(x)e^{isx}dx$, Then the function f(x)= $\frac{1}{\sqrt{2\pi}}\int_{-\infty}^{\infty}F[f(x)]e^{-isx}ds$

10.What is the Fourier Transforms of f(x-a) the Fourier Transforms of f(x) is F(s)?

Given that F[f(x)]=F(s)

i.e,F[f(x-a)]= $e^{ias}F(s)$

11. Define Fourier sine transform?

Fourier **sine** transform of f(x) is

$F_s[f(x)] = \sqrt{\frac{2}{\pi}}\int_0^{\infty} f(x)\sin sx dx$

12. Define Fourier sine transform its inverse?

Fourier **sine** transform o its **inverse** f(x) is

$$f(x)=\sqrt{\frac{2}{\pi}}\int_0^\infty F_s[f(x)]\sin sxds$$

13. Define Fourier cosine transform?

Fourier **cosine** transform of f(x) is

$$F_c[f(x)]=\sqrt{\frac{2}{\pi}}\int_0^\infty f(x)\cos sxdx$$

14. Define Fourier cosine transform its inverse?

Fourier **cosine** transform o its **inverse** f(x) is

$$f(x)=\sqrt{\frac{2}{\pi}}\int_0^\infty F_c[f(x)]\cos sxds$$

15. Find the sine transform of e^{-x} ?

WKT $F_s[f(x)]=\sqrt{\frac{2}{\pi}}\int_0^\infty f(x)\sin sxdx$

Here f(x) = e^{-x}

$$F_s[e^{-x}]=\sqrt{\frac{2}{\pi}}\int_0^\infty e^{-x}\sin sxdx$$

$$=\sqrt{\frac{2}{\pi}}\frac{s}{s^2+1}$$

16. State the Fourier Transforms of the derivative of a function?

$F\left[\frac{d^n f(x)}{dx^n}\right]=(-is)^n F(s);$ where F(s)=F[f(x)].

17. Define convolution theorem for Fourier Transforms?

If F(s) and G(s) are the Fourier Transforms of f(x) and g(x) respect then the Fourier Transforms of the convolution of f(x) and g(x) is the product of their Fourier Transforms

i.e., **F[(f*g)]=F(s).G(s)**

18. Define linear property of Fourier Transforms?

Then the linear property is,

F[af(x)+bg(x)]=aF(s)+bG(s).

19. Define Shifting property of Fourier Transforms?

Then the Shifting property is,

(i) F[f(x-a)]= $e^{ias}F(s)$.

(ii)F[$e^{ias}f(x)$]=F(s+a).

20. Define Change of scale property of Fourier Transforms?

Then the Change of scale property is,

F[f(ax)]= $\frac{1}{a}F(\frac{s}{a}), a>0$

21. Define Modulation theorem?

Then the Modulation theorem is

$$F[f(x)\cos ax]=\frac{1}{2}[f(s+a)+f(s-a)] where f(s)=F[f(x)]$$.

UNIT-3

[Applications of PDE(Boundary Value Problems)]

1.Explain the initial and boundary value problem?

In ordinary differential equation , first we get the general solution which contains the arbitrary constant and then the initial value . This type of problem is called **initial value problem.**

2. Explain the method of separation of variables?

In this way,the solution of the PDE z is dependent variable x,y is independent variable is coverted in to the solution of ODE. This method is known as **method of separation of variables.**

3. The one dimensional wave equation is..?

The one dimensional wave equation is

i.e. $$\frac{\partial^2 u}{\partial t^2} = a^2 \frac{\partial^2 u}{\partial x^2}$$

4. The three possible solutions of $\frac{\partial^2 u}{\partial t^2} = a^2 \frac{\partial^2 u}{\partial x^2}$ are...?

Then the three possible solutions is $\frac{\partial^2 u}{\partial t^2} = a^2 \frac{\partial^2 u}{\partial x^2}$

$$(i)u(x,t) = (Ax + B)(Ct + D)$$

$$(ii)u(x,t) = (Ae^{px} + Be^{-px})(Ce^{pat} + De^{-pat})$$

$$(iii)u(x,t) = (A\cos px + B\sin px)(C\cos pat + D\sin pat)$$

5. The PDE of a vibrating string is $\frac{\partial^2 u}{\partial t^2} = a^2 \frac{\partial^2 u}{\partial x^2}$ what is a^2?

Then the vibrating string is $\frac{\partial^2 u}{\partial t^2} = a^2 \frac{\partial^2 u}{\partial x^2}$

$$a^2 = \frac{T}{m} = \frac{Tension}{mass}$$

6. Explain the various variables involved in one dimensional wave equation ?

The one dimensional wave equation is $\frac{\partial^2 u}{\partial t^2} = a^2 \frac{\partial^2 u}{\partial x^2}$.Here x and t are the variables .Where x denotes length and t denotes time .

7. Define temperature gradient?

This rate of changes of temperature w.r.to distance is called the temperature gradient and denoted by $\frac{\partial u}{\partial x}$.

8. Define steady state temperature distribution?

If the temperature will not change when time varies is called steady state temperature distribution.

9. How many boundary conditions required to solve completely $\frac{\partial^2 u}{\partial t^2} = a^2 \frac{\partial^2 u}{\partial x^2}$?

Then the three conditions

10. State the law assumed to derive the one dimensional heat equation?

(i) Heatflows a higher temperature to lower temperature .

(ii) To produce temperature change in a body is proportional to the mass of the body and to the temperature change .

(iii) An area is proportional to the area and to the temperature gradient normal to the area.

11. What is the basic difference between the solutions of one dimensional wave equation and one dimensional heat equation ?

The correct solution of one dimensional wave equation is of periodic in nature. But solution of heat flow equation is not in periodic in nature.

12. Give three possible solutions of the equation $\frac{\partial u}{\partial t} = a^2 \frac{\partial^2 u}{\partial x^2}$?

The three possible solutions is

$(i) u(x,t) = (Ax + b)$

$(ii) u(x,t) = e^{\alpha^2 p^2 t}(Ae^{px} + Be^{-px})$

$(iii) u(x,t) = e^{-\alpha^2 p^2 t}(A\cos px + B\sin px)$

13. State Fourier law of heat conduction?

The rate at which heat flows across an area A at a distance x from one end of a bar is given by $Q = -KA\left(\frac{\partial u}{\partial x}\right)_x$ **,** k is thermal conductivity and $\left(\frac{\partial u}{\partial x}\right)_x$ means the temperature gradient at x.

14. Write the solution of one dimensional heat equation. When the time derivative is absent?

When time derivative is absent is the heat flow equation is $\frac{\partial^2 u}{\partial x^2} = 0$.

15. In steady state,two dimensional heat equation in cartesian coordinates is..?

Then the steady state,two dimensional heat equation in cartesian coordinates

$$\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = 0$$

16. Write the boundary condition of the string equation ,to initial displacement f(x) and initial velocity g(x)?

Then the boundary condition are

$(i) y(0,t) = o for all t > 0$

$(ii) y(l,t) = 0 for all t > 0$

$(iii) \frac{\partial y(x,0)}{\partial t} = g(x) for all x in (0,l)$

$(iv) y(x,0) = f(x) for all x in (0,l)$

17. Write the boundary condition of string equation ,to non zero initial velocity?

Then the boundary condition are non zero initial velocity is

$(i)\, y(0,t) = o for all t > 0$

$(ii)\, y(l,t) = 0\, for all t > 0$

$(iii)\, y(x,0) = 0\, for all x in (0,l)$

$(iv)\, \frac{\partial y(x,0)}{\partial t} = g(x)\, for all x in (0,l)$

18.Explain the term steady state?

When the heat flow is independent of time "t", it is called steady state. In steady state the heat flow is only w.r.to the distance "x".

19. Obtain one dimensional heat flow equation from two dimensional heat flow for unsteady case?

When unsteady state condition exists the two dimensional heat flow equation is given by,

$$\frac{\partial u}{\partial t} = a^2\left(\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2}\right)$$

In one dimensional heat flow equation is given by,y-direction and hence $\frac{\partial^2 u}{\partial y^2} = 0$.

Then the heat flow equation is $\frac{\partial u}{\partial t} = a^2 \frac{\partial^2 u}{\partial x^2}$.

20.What is meant by two dimensional heat flow?

The heat flows in xy- direction.

21.Explain the term thermally insulated ends?

If there will be no heat flow passes through the ends of the bar then that two endsare said to be thermally insulated.

UNIT –IV

[Partial Differential Equation]

1. Find the order of a PDE?

The order of a PDE is the order of the highest partial derivative occurring in the equation .

2. Find the formation of PDE?

(i) By elimination of arbitrary constants.

(ii) By elimination of arbitrary functions.

3.Explain the formation of PDE by elimination of arbitrary constants?

Let f (x, y, z, a, b) = 0………………….(1)

Be an equation which contains two arbitrary constant "a"and "b".PDE (1) w.r.to "x" and "y" we get two more equations.

4.From the PDE eliminating the arbitrary constants from $z=(x-a)^2+(y-b)^2+1$?

Given $z=(x-a)^2+(y-b)^2+\mathbf{1}$ ....(1)

$p=\frac{\partial z}{\partial x}=2(x-a)$

$q=\frac{\partial z}{\partial y}=2(y-b)$ ....(2) and (3)

Substituting (2) and (3) in (1)we get

$$z=\frac{p^2}{4}+\frac{q^2}{4}+1$$

5. From the PDE eliminating arbitrary constants a and b from $z=(x+a)(y+b)$?

Given $z=(x+a)(y+b)$ ....(1)

$p=\frac{\partial z}{\partial x}=y+b$

$q=\frac{\partial z}{\partial y}=x+a$ ....(2) and (3)

Substituting (2) and (3) in (1) we get $z=pq$

6. From the PDE eliminating arbitrary constants a and b from $z=ax+by+ab$?

Given $z=ax+by+ab$ .... (1)

$p=\frac{\partial z}{\partial x}=a$

$q=\frac{\partial z}{\partial y}=b$ .... (2) and (3)

Substituting (2) and (3) in (1) we get $z=px+qy+pq$

7. From the PDE eliminating arbitrary constants a and b from $z=ax+by+a^2+b^2$?

Given $z=ax+by+a^2+b^2$ .... (1)

$p=\frac{\partial z}{\partial x}=a$

$q=\frac{\partial z}{\partial y}=b$ .... (2) and (3)

Substituting (2) and (3) in (1) we get $z=px+qy+p^2+q^2$

8.Eliminate the function "f" from $z = f(x^2 + y^2)$?

Given $z = f(x^2 + y^2)$ .... (1)

$$p = \frac{\partial z}{\partial x} = f'(x^2 + y^2)2x$$
$$q = \frac{\partial z}{\partial y} = f'(x^2 + y^2)2y$$
......(2) and (3),

$$f' = \frac{p}{2x}$$
$$f' = \frac{q}{2y}$$
.........(4) and (5)

Substituting (2) and (3) in (1) we get $\frac{p}{2x} = \frac{q}{2y} (or) py = qx$.

9. Define the complete integral?

A solution in which the number of arbitrary constant is equal to the number of independent variable is called complete integral or complete solution.

10. Define the particular integral?

In complete integral if we give particular values to the arbitrary constant we get particular integral.

11. Define the Singular integral?

Let f (x, y, z, p, q) = 0 be a PDE whose complete integral is φ(x, y, z, a, b)...................................(1)
Diff .P.w.r.to "a" and "b" and then equal to zero , we get

$$\frac{\partial \phi}{\partial a} = 0$$
$$\frac{\partial \phi}{\partial b} = 0$$

The eliminate of 'a' and 'b' from the three equations is called singular integral.

12. Solve $\frac{\partial^2 z}{\partial x^2} = \sin x$.

Given $\frac{\partial^2 z}{\partial x^2} = \sin x$.

$$\frac{\partial z}{\partial x} = -\cos x + f(y)$$
$$z = -\sin x + xf(y) + g(y)$$

13. Solve $\frac{\partial^2 z}{\partial x^2} = xy$

Given that $\frac{\partial^2 z}{\partial x^2} = xy$.

$$\frac{\partial z}{\partial x} = y\frac{x^2}{2} + f(y)$$
$$z = y\frac{x^3}{6} + xf(y) + g(y)$$

14. Solve $\frac{\partial^2 z}{\partial x \partial y} = \sin x$**.**

Given that $\frac{\partial^2 z}{\partial x \partial y} = \sin x$**.**

$$\frac{\partial z}{\partial y} = -\cos x + f(y)$$

$$z = -y\cos x + f(y) + g(x)$$

15. From the PDE eliminating arbitrary a and b from $z = a(x+y)+b$**?**

Given $z = a(x+y)+b$ .... (1)

$$p = \frac{\partial z}{\partial x} = a$$

$$q = \frac{\partial z}{\partial y} = a$$
.... (2) and (3)

Substituting (2) and (3) in (1) we get $p = q$.

16. Write the complete integral of $z - px = qy + pq$ **?**

Given $z - px = qy + pq$

Then we know that $z = px + qy + pq$

This is of Clairaut's type Hence replace p by a and q by b in the complete integral is $z = ax + by + ab$

17. Write the complete integral of $z = px + qy + \sqrt{pq}$?

Given $z = px + qy + \sqrt{pq}$

Then we know that $z = px + qy + \sqrt{pq}$

This is of Clairaut's type Hence replace p by a and q by b in the complete integral is $z = ax + by + \sqrt{ab}$

18. Write the complete integral of $z = px + qy + \sqrt{1 + p^2 + q^2}$ **?**

Given $z = px + qy + \sqrt{1 + p^2 + q^2}$.

This is of Clairaut's type Hence replace p by a and q by b in the complete integral is $z = ax + by + \sqrt{1 + a^2 + b^2}$.

19. Write the general solution of non-homogeneous linear PDE?

The general solution of non-homogeneous linear PDE

If f(D,D')z=F(x ,y) is z= $\sum C_1 e^{hx + f_1(h)y} + \sum C_2 e^{hx + f_2(h)y} + \ldots\ldots\ldots\ldots$

20. Find the singular integral of z=px+qy+pq?

Given that the complete integral is z=ax+by+ab.........(1).

$$\frac{\partial z}{\partial x} = x + b = 0 \Rightarrow b = -x$$

$$\frac{\partial z}{\partial y} = y + a = 0 \Rightarrow a = -y$$

...........................(2) and (3).

UNIT –V

[Z-Transform and Difference Equation]

1. Define the Z-Transform?

Consider the sequence is f(n)=f(0),f(1),f(2),f(3),......................f(n). Then for all positive integer n=0,1,2,3,..............∞.Then the Z- Transform of {f(n)} is defined as

$$Z\{f(n)\} = \sum_{n=0}^{\infty} f(n) z^{-n}$$

2. Define the initial value theorem?

Then the initial value theorem is

If
$$Z[f(n)] = F(z)$$
$$then \lim_{z\to\infty} F(z) = f(0) = \lim_{t\to 0} f(0)$$

3. Define the Final value theorem?

Then the Final value theorem is

If
$$Z[f(n)] = F(z)$$
$$then \lim_{t\to\infty} F(t) = \lim_{t\to 1} (z-1)F(z)$$

4. Define the linear property ?

Then the linear property is

$$Z[af(n) + bg(n)] = aF(z) + bG(n)$$
$$where Z[f(n)] = F(Z)$$
$$and Z[g(n)] = G(z)$$

Where a,b are constants.

5. Define the first shifting property ?

Then the first shifting property is ,If
$$Z[f(t)] = F(z), then$$
$$Z[e^{-at} f(t)] = F[ze^{aT}]$$

6. Define the inverse Z-transform?

If $Z[f(k)] = F(z)$, then the inverse Z-transform is

$$z^{-1}[f(z)] = F(k)$$

(or)

If
$$Z[f(n)] = F(z), then$$
$$Z^{-1}[F(z)] = f(n)$$

7. Define the method of partial fraction?

To find inverse transform of a function F(z) by using partial fraction method, it is convenient to write F(z) as $\frac{F(z)}{z}$ and then split into partial fraction.

8. Find the inverse Z-transform using Residue theorem?

If $Z[f(n)] = F(z)$, then f(n) which gives the inverse Z-transform of F(z) is obtained the result

$$f(n) = \frac{1}{2\pi i} \int_c z^{n-1} F(z) dz$$

Where C is the closed contour which encloses all the poles of the integrand.

9. Define the convolution of two sequences ?

The convolution of two sequences $\{ f(n) \}$ and $\{ g(n) \}$ is defined as

$$[\, f(n) * g(n) \,] = \sum_{r=0}^{n} f(r) g(n-r) \quad \text{[For right sided sequence]}$$

(or)

$$[\, f(n) * g(n) \,] = \sum_{r=-\infty}^{\infty} f(r) g(n-r) \quad \text{[For two sided or bilateral sequence]}$$

10. Define the convolution theorem ?

Then the convolution theorem is,

(i) $Z[f(n)*g(n)] = F(z).G(z), where$

$Z[f(n)] = F(z) and Z[g(n)] = G(z)$

(ii) $Z[f(t)*g(t)] = F(z).G(z), where$

$Z[f(t)] = F(z) and Z[g(t)] = G(z)$

11. Find $Z\left[\frac{a^n}{n!}\right]$ in Z-transform?

We know that $Z\left[\frac{a^n}{n!}\right] = \sum_{n=0}^{\infty} \frac{a^n}{n!} z^{-n}$

$$= \sum_{n=0}^{\infty} \frac{(az^{-1})^n}{n!} = 1 + \frac{az^{-1}}{1!} + \frac{(az^{-1})^2}{2!} + \ldots\ldots\ldots$$

$$= e^{az^{-1}}$$

$$Z\left[\frac{a^n}{n!}\right] = e^{\frac{a}{z}}$$

12. Find $Z[ze^{-iat}]$ using Z- transform?

We know that $Z[ze^{-iat}] = Z[ze^{-iat}.1]$

$$= \{z(1)\}_{z \to ze^{iat}}$$

$$= \left[\frac{z}{z-1}\right]_{z \to ze^{iat}}$$

$$Z[ze^{-iat}] = \left[\frac{ze^{iat}}{ze^{iat}-1}\right]$$

13. Find $Z[a^n]$ using Z- transform?

We know that $Z[a^n] = \sum_{n=0}^{\infty} a^n z^{-n}$

$$= \sum_{n=0}^{\infty} \left(\frac{a}{z}\right)^n$$

$$= \frac{1}{1-\frac{a}{z}} = \frac{z}{z-a} \; if\, |z| > |a|$$

14. Find $Z[a^{n-1}]$ using Z- transform?

We know that $Z[a^{n-1}] = \sum_{n=0}^{\infty} a^{n-1} z^{-n}$

$$= Z^{-1} \frac{z}{z-a}$$

$$= \frac{1}{z-a} if |z| > |a|$$

15. Write the damping rate for Z- transform?

Then the damping rate for Z- transform is

$$Z\{f(n)\} = \bar{f}(z) = F(z), then$$

If $(i) Z\{a^{-n} f(n)\} = \bar{f}(az) = F(az)$

$$(ii) Z\{a^{-n} f(n)\} = \bar{f}\left(\frac{z}{a}\right) = F\left(\frac{z}{a}\right)$$

16. Find $Z[n]$ using Z- transform ?

We know that $Z[n] = \sum_{n=0}^{\infty} n z^{-n}$

$$= \frac{1}{z} + \frac{2}{z^2} + \frac{3}{z^3} + \ldots\ldots\ldots\ldots$$

$$= \frac{1}{z}\left[1 + \frac{2}{z} + \frac{3}{z^2} + \ldots\ldots\ldots\ \ldots\ldots\right]$$

$$= \frac{1}{z}\left[1 - \frac{1}{z}\right]^{-2} = \frac{1}{z}\left[\frac{z-1}{z}\right]^{-2} = \frac{z}{(z-1)^2}$$

17. Define the second shifting property ?

Then the second shifting property is ,

$$Z[f(t)] = F(z), then$$

If $(i) Z[f(t+T)] = zF(z) - zf(0)$

$$(ii) Z[f(t+kT)] = Z[f(n+k)T]$$

18. Find the Z-transform of coshnØ.

We know that Z{cosnØ} =