CHAPTER 2

FLUID STATICS

What is hydrostatics?

Hydrostatics is the study of fluid bodies that are

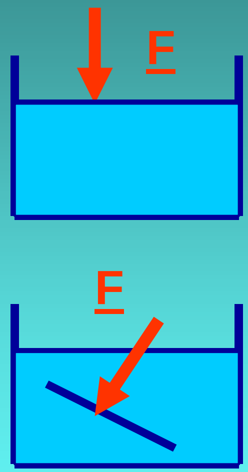
- At rest
- Moving sufficiently slowly so there is no relative motion between adjacent parts of the body

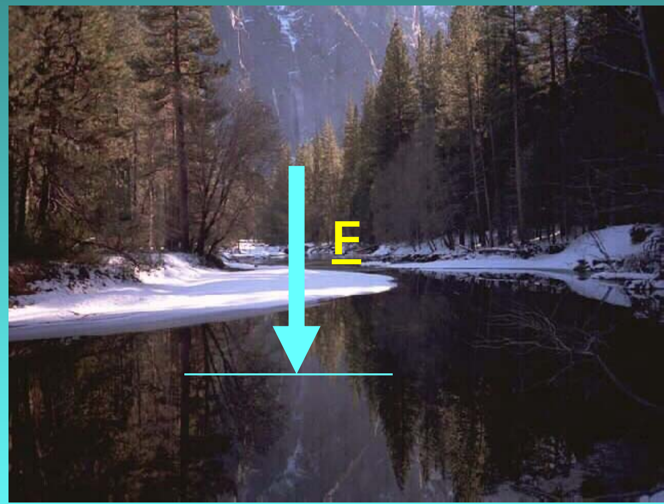
For hydrostatic situations

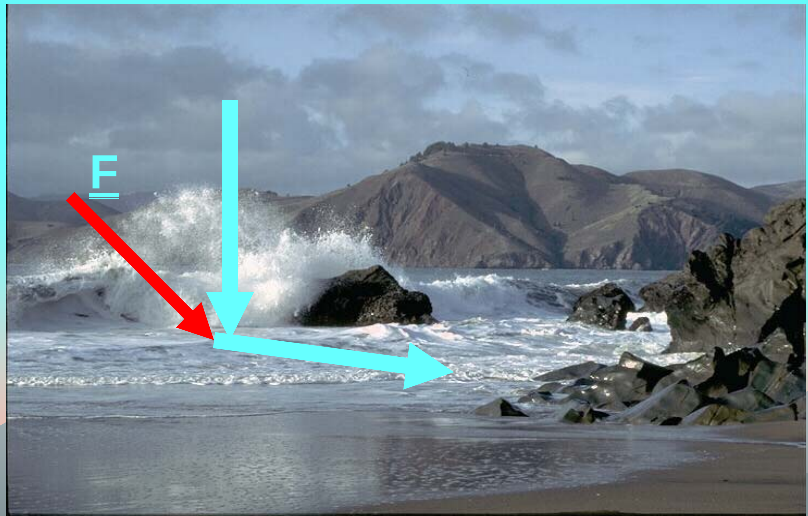
- There are no shear stresses
- There are only pressure forces that act perpendicular to any surface

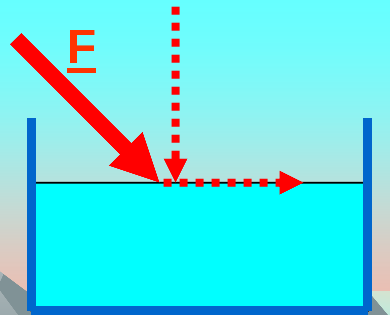
http://1stcivilfluidmechanics.webs.com/

Static

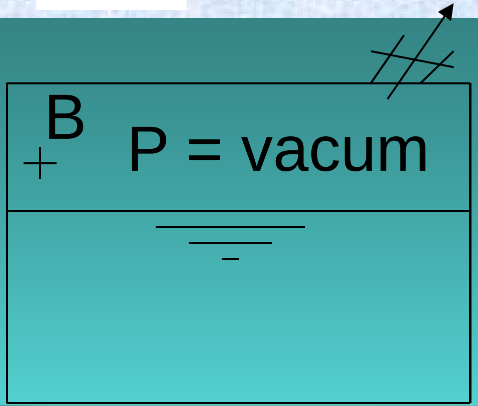


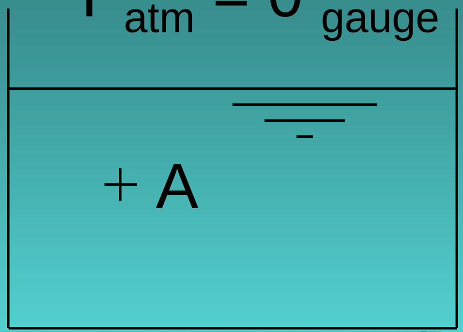


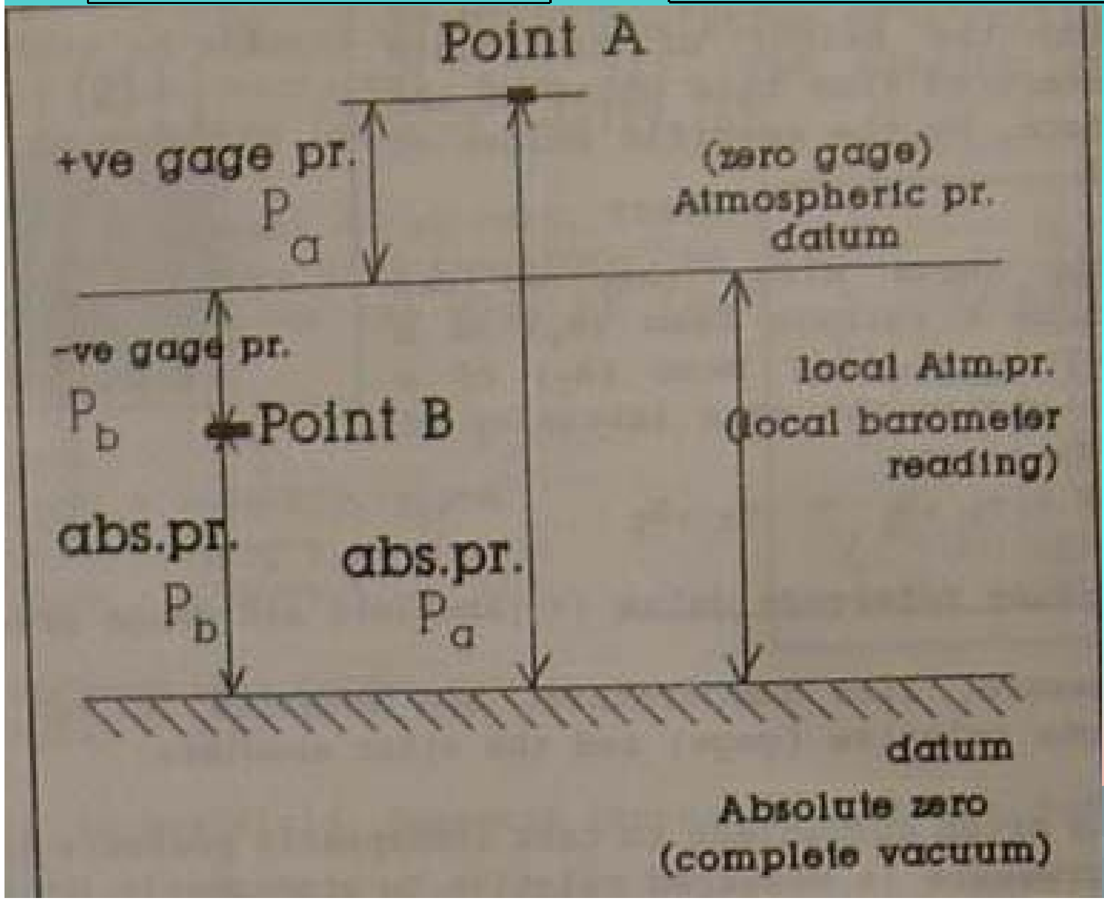




Dynamic

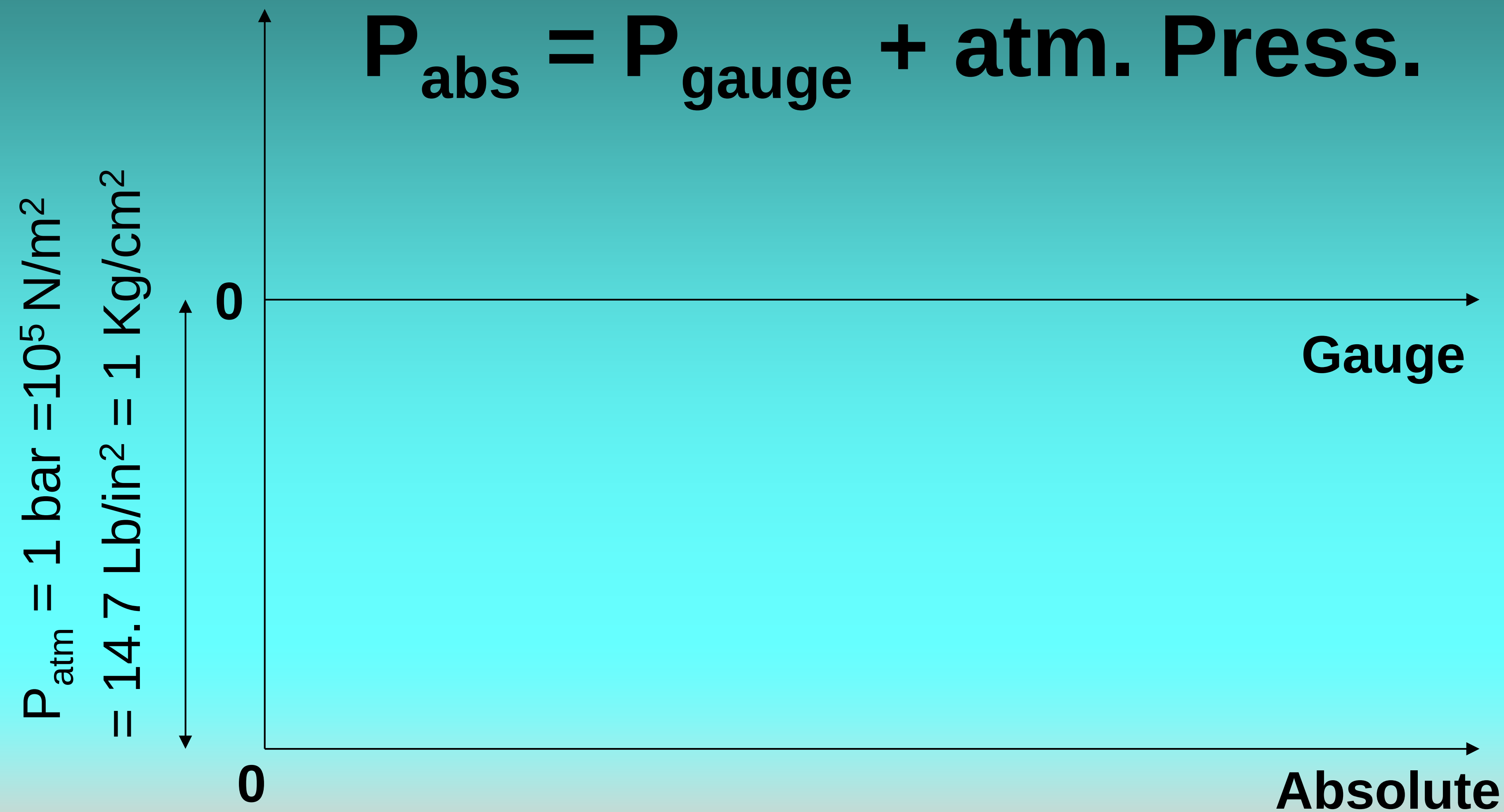






Barometer mercury column hg

$P_{abs} = P_{gauge}$ + atm. Press.



- The science of fluid static will be treated in two parts:
  - The study of pressure and its variation throughout a fluid.
  - The study of pressure forces on finite surfaces.

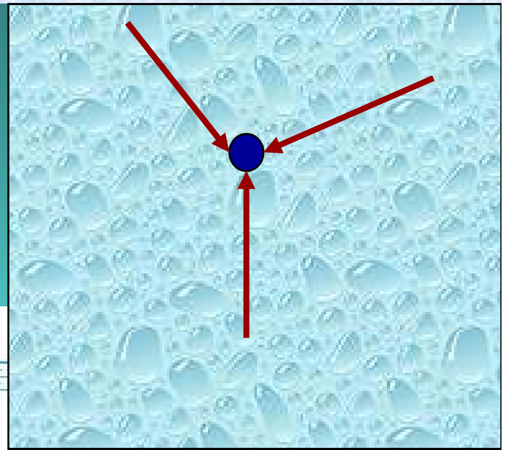
- Since there is no motion of a fluid layer, there are no shear stresses in the fluid. There is only normal pressure forces acting on the surfaces of the free bodies

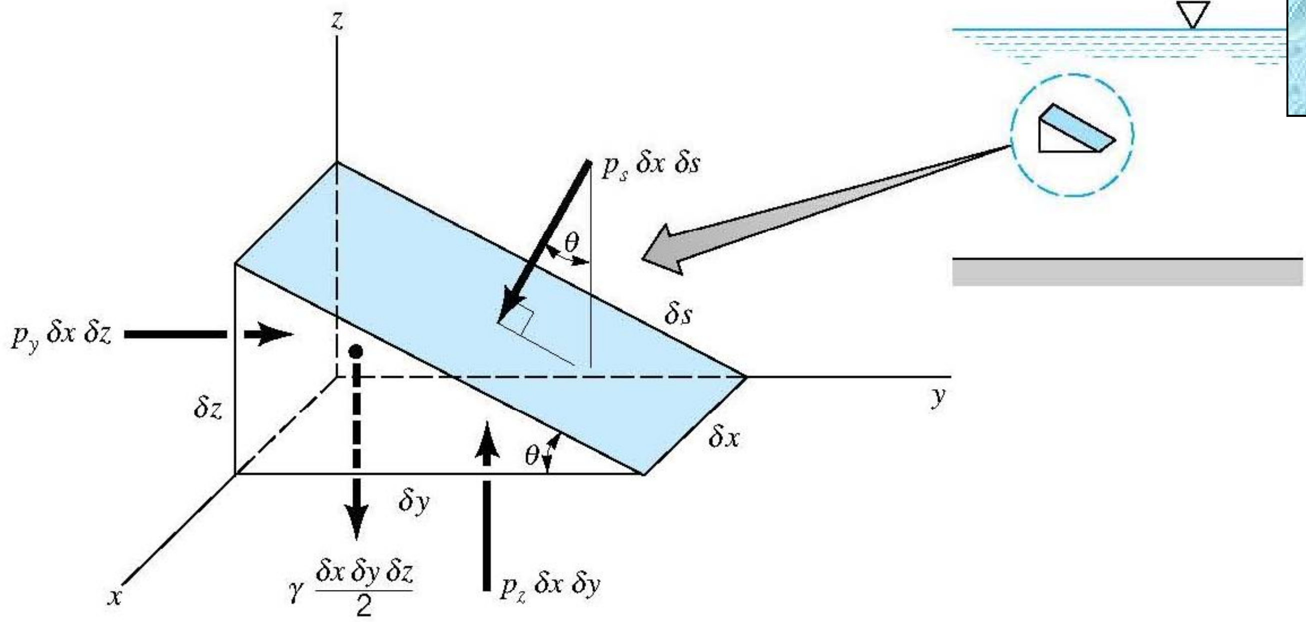
The objective of the study

- Establish a relation for pressure variation along a vertical depth in fluid.
- Study the fluids in Relative equilibrium.
- Compute the hydrostatic forces and pressure distribution on submerged bodies.
- Analyze the stability of floating bodies.

Principles of Fluid Static

- Pressure at a point





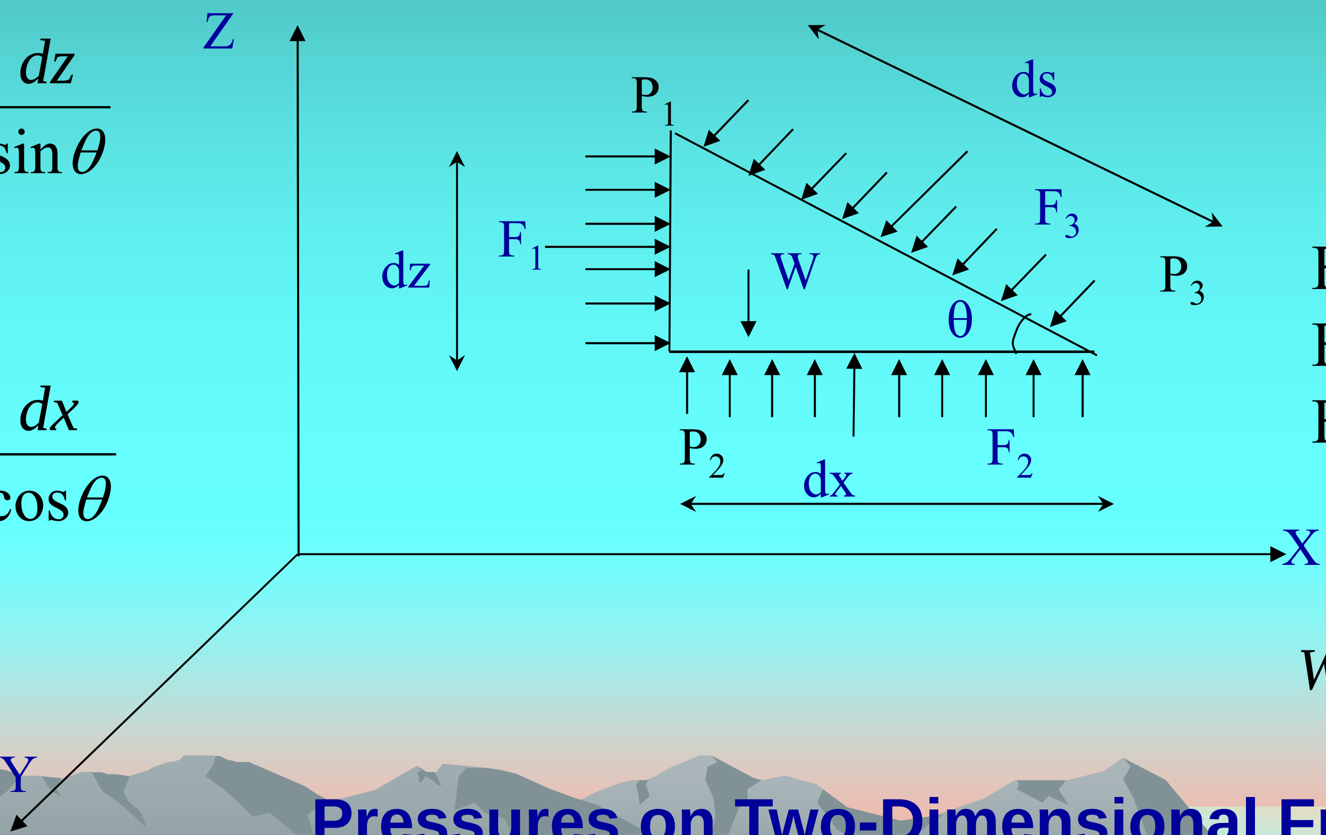
As fluids at rest cannot contain shearing stresses, pressure must be shown acting inward upon the free body.

Consider a conventional two-dimensional free body of fluid having unit width normal to the plane of paper.

$$ds = \frac{dz}{\sin\theta}$$

and

$$ds = \frac{dx}{\cos\theta}$$



$$F_1 = P_1 \,.\, dz$$
$$F_2 = P_2 \,.\, dx$$
$$F_3 = P_3 \,.\, ds$$

$$W = \frac{1}{2}\gamma\, dz\, dx$$

Pressures on Two-Dimensional Free Body

The forces acting on this elemental volume are:

$$F_1 = P_1 . dz \qquad F_2 = P_2 . dx \qquad F_3 = P_3 . ds$$

and

$$W = \frac{1}{2}\gamma\, dz\, dx$$

From statical equilibrium

$$\sum F_x = 0 \Rightarrow F_1 - F_3 \sin\theta = 0$$

$$= P_1\ dz - P_3 \frac{dz}{\sin\theta} \times \sin\theta = 0$$

$$P_1 = P_3$$

$$\sum F_z = 0$$

$$\therefore P_2\,dx - \frac{1}{2}\gamma\,dz\,dx - P_3\frac{dx}{\cos\theta}\cos\theta = 0$$

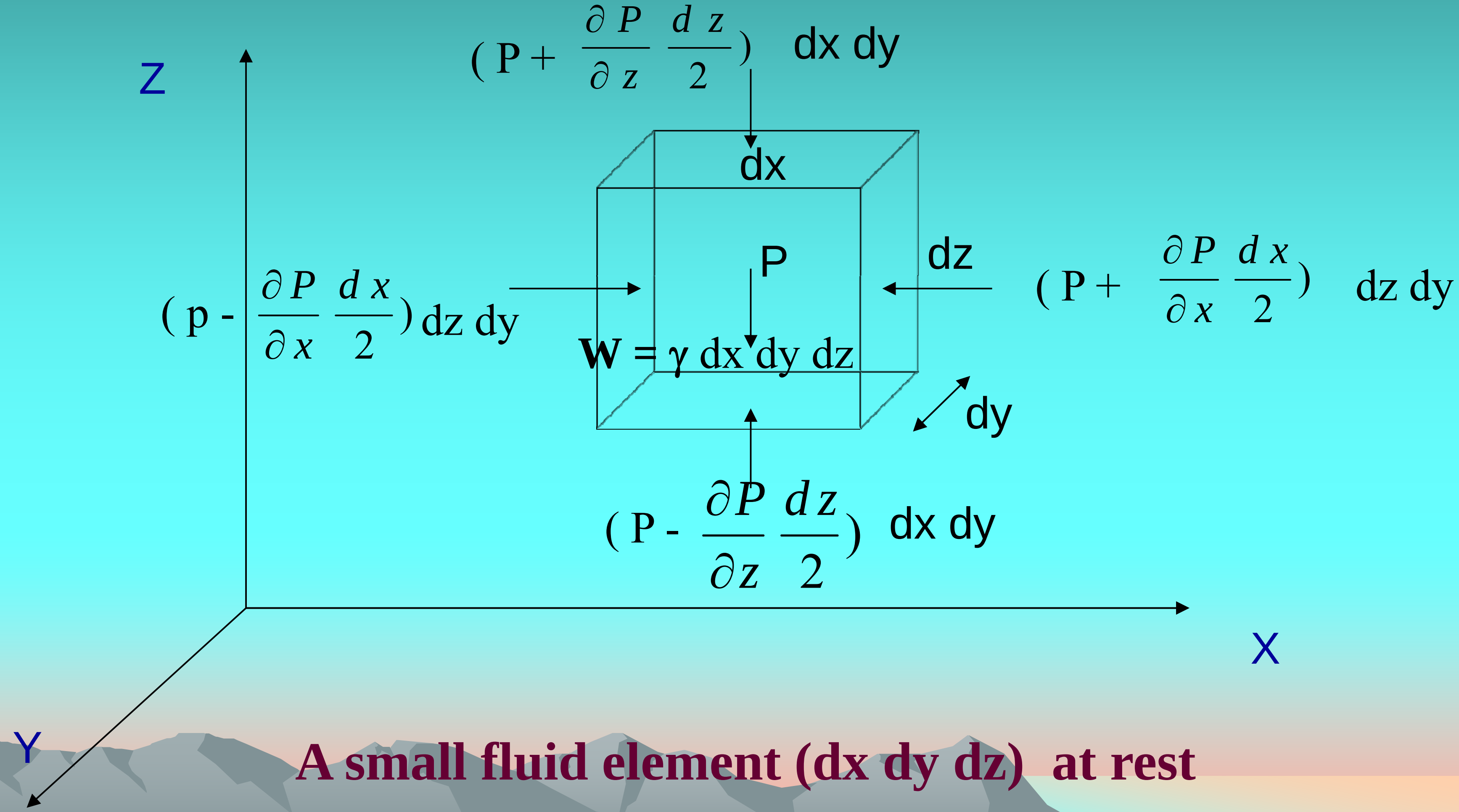
$$\therefore P_2 = P_3 + \frac{1}{2}\gamma\,dz$$

dz becomes zero $\Rightarrow$ P3 = P2 .

$$\boxed{\therefore \quad P_1 = P_2 = P_3}$$

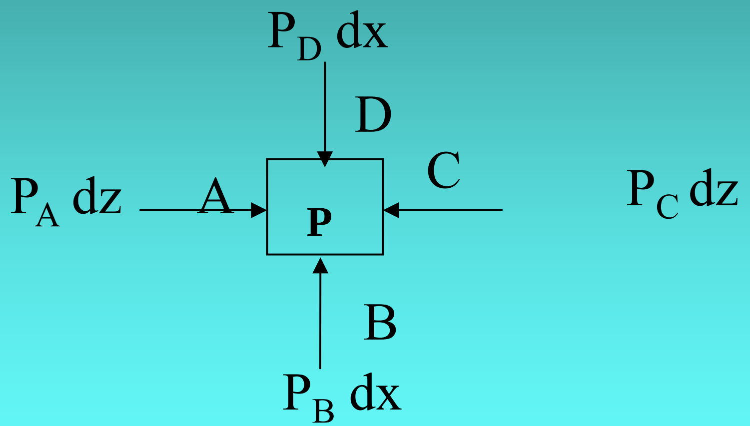
- The equation (p1 = p2 = p3) indicates that the pressure at a point is the same in all direction. This is known as ***Pascal's law*** and applies to any fluid at rest.

▪Pressure Distribution in a fluid at rest



A small fluid element (dx dy dz) at rest

By considering the static equilibrium of one differential two-dimensional element has a unit width. i.e. (dy=1)



$$\Sigma F_x = P_A dz - P_C dz = 0 \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \; Eq.\,\mathrm{I}$$

$$\Sigma F_z = P_B dz - dw - P_D dx = 0 \quad \ldots\ldots\ldots\ldots \; Eq.\,\mathrm{II}$$

Substitute in the first equation (x-direction) :

$$\left(P - \frac{\partial P}{\partial x}\frac{dx}{2}\right)dz - \left(P + \frac{\partial P}{\partial x}\frac{dx}{2}\right)dz = 0$$

$$\frac{\partial P}{\partial x} = 0$$

This means that all points at the same elevation within a fluid have the same pressure value.

Substitute in second equation (z-direction) :

$$\left(P - \frac{\partial P}{\partial z}\frac{dz}{2}\right) dx - \gamma\ dz\ dx - \left(P + \frac{\partial P}{\partial z}\frac{dz}{2}\right) dx = 0$$

$$-\frac{\partial P}{\partial z}\ dz\ dx - \gamma\ dz\ dx = 0$$

$$\frac{\partial P}{\partial z} = -\gamma$$

Therefore, pressure is a function only of z and the total derivative replaces the partial derivative.

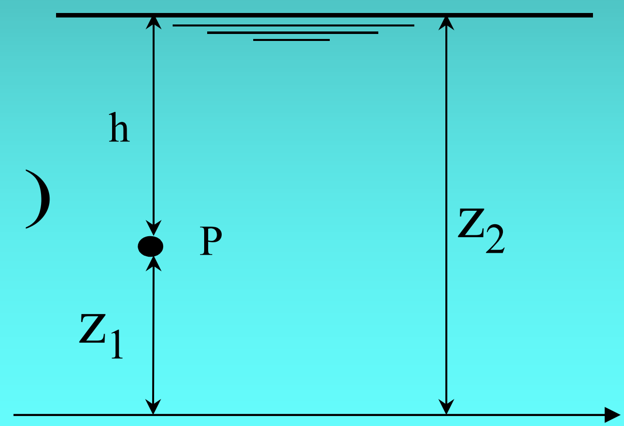
$$\frac{dP}{dz} = -\gamma$$

By integrating the previous equation we obtain

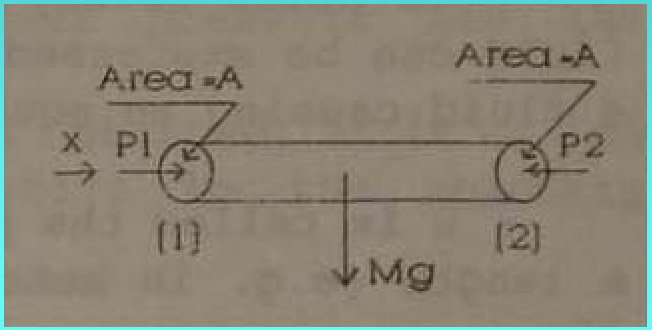
$$\int_{0}^{P} dP = -\gamma \int_{Z_2}^{Z_1} dZ$$

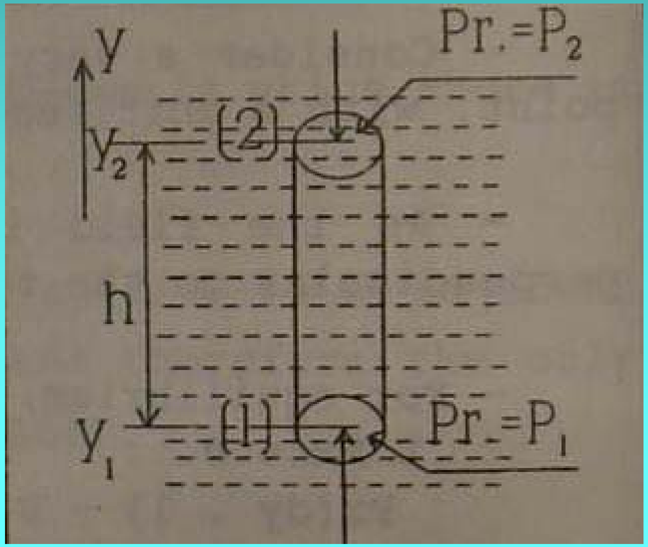
$$P = \gamma \left(Z_2 - Z_1 \right)$$

$$P = \gamma \ h$$



Which is the basic equation of fluid statics, where pressure in the vertical direction is proportional to the depth of the liquid.

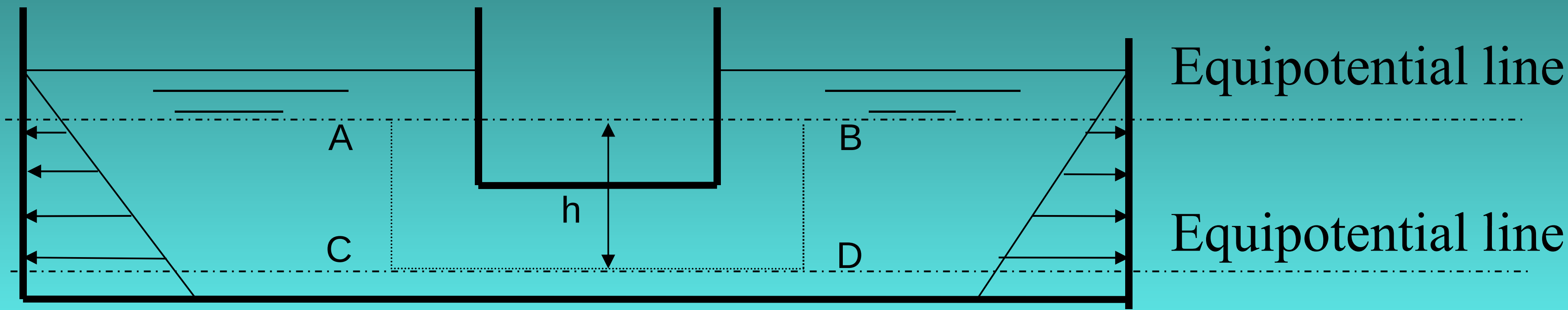

Area =A
Area =A
x
P1
P2
(1)
(2)
Mg


y
Pr.=P2
y2
(2)
h
y1
(1)
Pr.=P1

Conclusion:

- Any two points at the same elevation in a continuous length of the same liquid are at the same pressure. (one equi-potential surface)
- Pressure increases as one goes down in a liquid column (remember the pressure changes on diving into a swimming pool).
- Pressure variation in air is neglected in relatively small distance (γ_a is small w.r.t. γ_w)

Equality of pressures in a continuous body of fluid



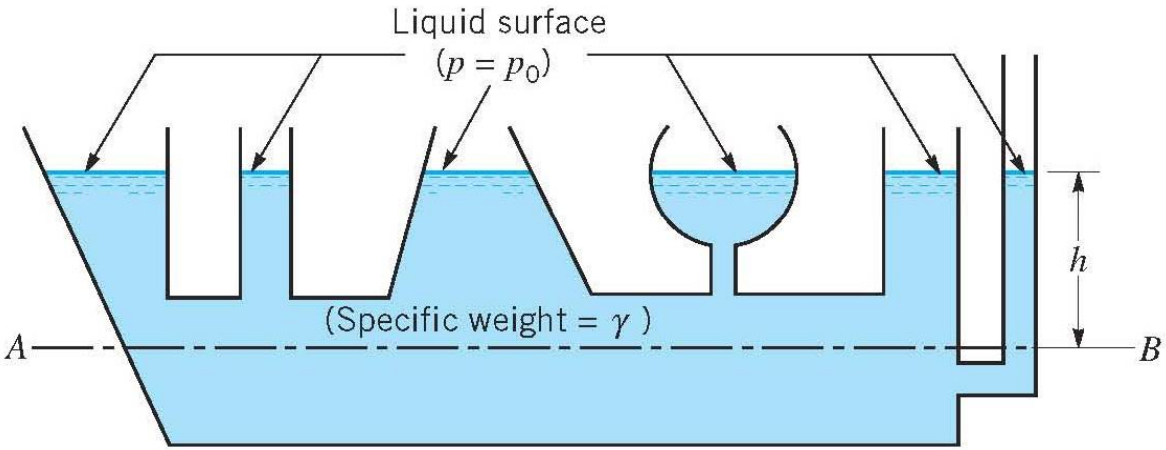
PA = PB , PC = PD , and PC = PA + γ h

<u>Specifications of Equipotential Line (surface)</u> :-

-Fluid must be at rest.

-Same fluid

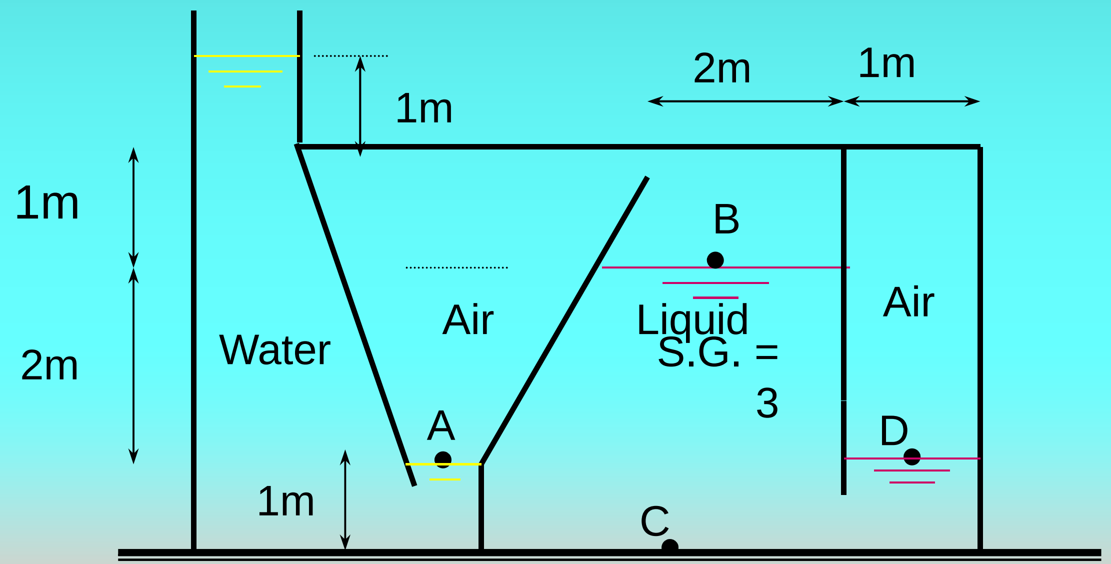
-Continuity of fluid



Example:

Determine the pressure at A, B, C and D in K Pa.

$$\left(\gamma_w = 9.81\,KN/m^3 = 9.81\,Kpa/\,m\right)$$



Solution:

$$P_A = \gamma_w (4\,m) = \underline{\underline{39.24\ KPa}}$$

$$P_B = P_A$$

$$P_C = P_B + (3\gamma_w) \times 3 = \underline{\underline{127.53\ KPa}}$$

$$P_D = P_B + (3\gamma_w) \times 2 = \underline{\underline{98.10\ KPa}}$$

$$or = P_C - (3\gamma_w) \times 1 = \underline{\underline{98.10\ KPa}}$$

Pressure expressed as height of fluid *(Pressure Head)*

We can express pressure in terms of height of a column of fluid.

$$h = \frac{P}{\gamma}$$

When pressure is expressed in this way, it is commonly referred to as pressure head.

Example:

If a pressure in a tank is 345 KPa, find the equivalent pressure head of :

1- Water.

2- Mercury.

3- Heavy fuel oil with a specific gravity of 0.92

Solution:

$$h = \frac{P}{\gamma}$$

$$h_w = \frac{345}{9.81} \frac{KN/m^2}{KN/m^3} = \underline{\underline{35.2\ m\ of\ water}}$$

$$h_{Hg} = \frac{345}{13.6 \times 9.81} \frac{KN/m^2}{KN/m^3} = \underline{\underline{2.59\ m\ of\ mercury}}$$

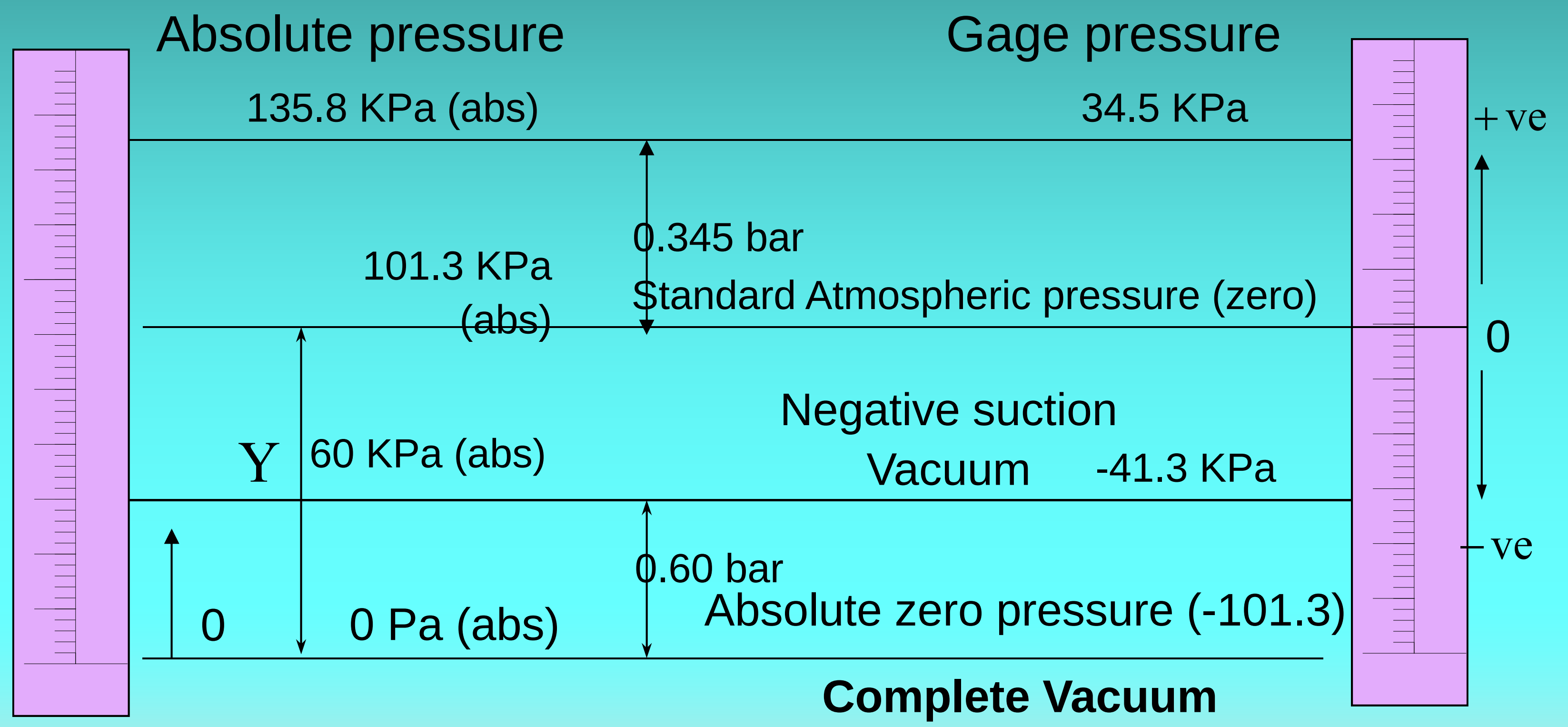
$$h_{oil} = \frac{345}{0.92 \times 9.81} \frac{KN/m^2}{KN/m^3} = \underline{\underline{38.27\ m\ of\ oil}}$$

Pressure Scales

- The usual datums are *absolute zero* and *local atmospheric pressure.*

- ***absolute pressure***: When a pressure is expressed as a difference between its value and a complete vacuum.

- ***gage pressure:*** When it is expressed as a difference between its value and *the local atmospheric pressure.*



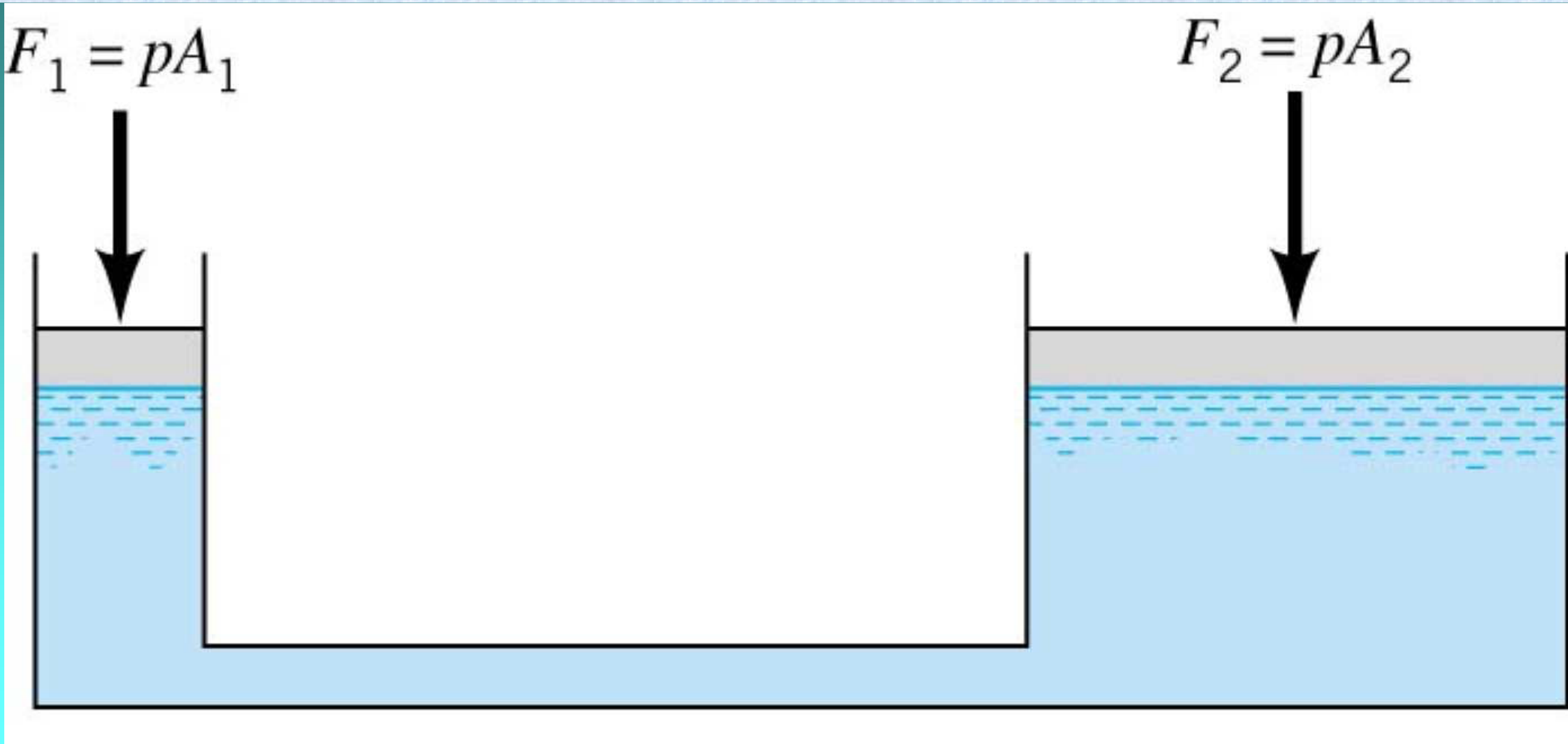
Y = 10.33 m of Water = 760 mm of Hg = 1 bar , 1 bar = 10^5 Pascal

Pressure Transmission in fluids

- ***Pascal's Principle:*** If a pressure is applied to any point in an enclosed fluid or system of enclosed fluids, it is transmitted without loss in all directions throughout the fluid system.

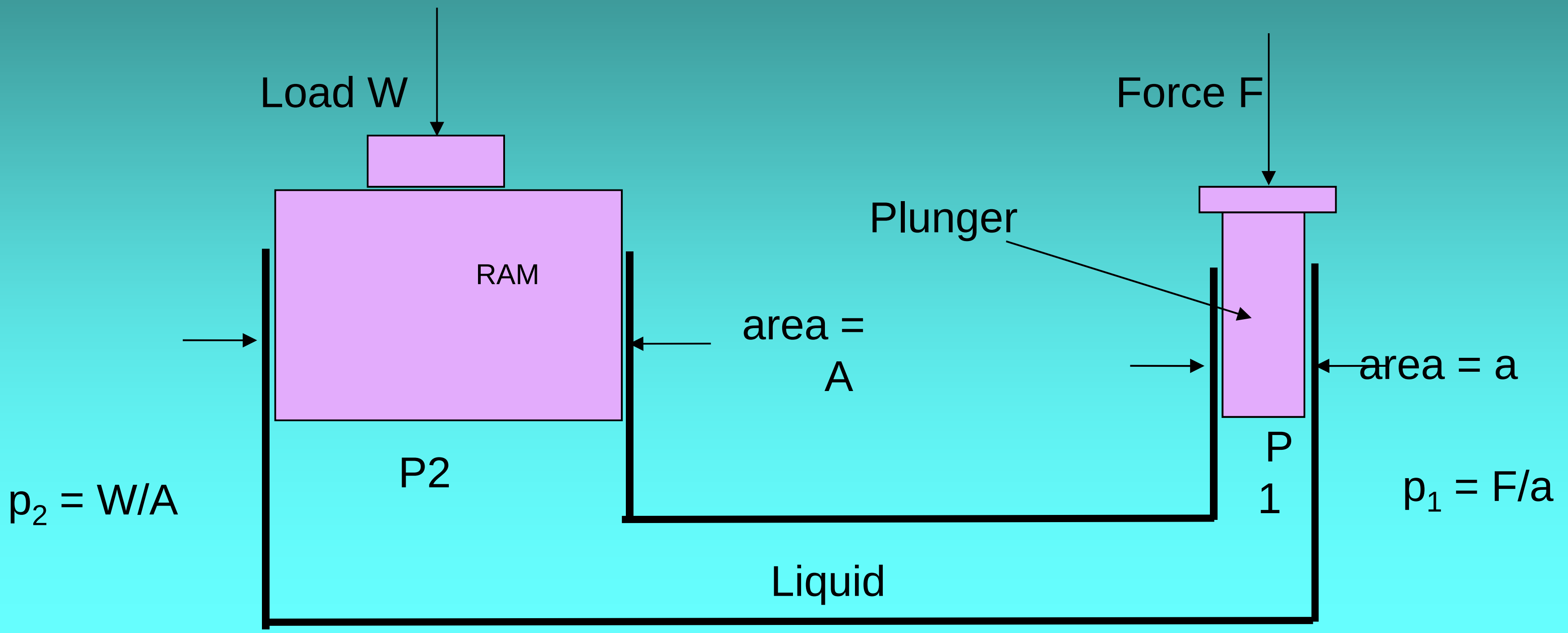
- ***Pascal's law:*** states that 'Intensity of pressure is transmitted equally in all directions through a mass of fluid at rest.

- ***Application of Pascal's law:*** hydraulic press, hydraulic jack, hydraulic lift and hydraulic crane.



p(piston1) = p(piston2) (if at equilibrium).

- p = p(piston1) + h . The pressure from the piston head is transmitted to all points in the fluid.
- F1/A1 = F2/A2 . A small force applied to a small area can be used to apply a large force to a large area.



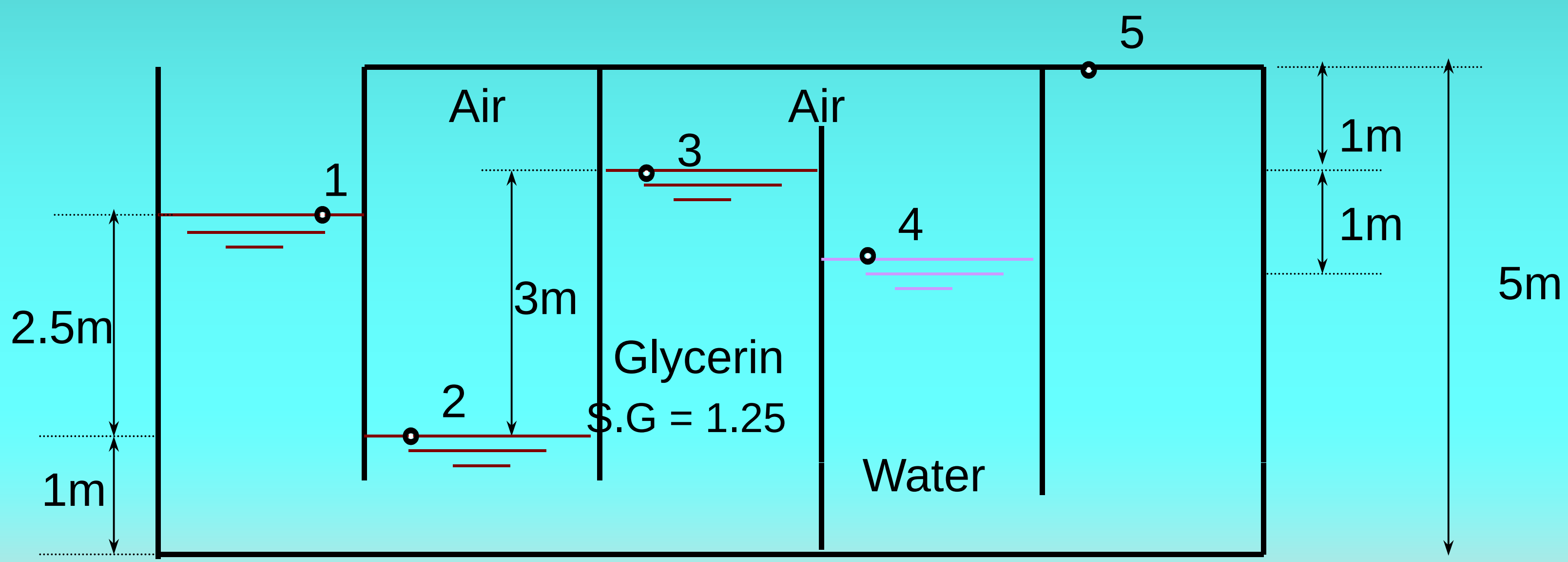
Working principle of hydraulic press

$$p_1 = p_2 \longrightarrow W = F\ (A/a)$$

- A large load can be lifted by the ram by applying a small force F on the plunger.

Example:

A device contains cells of air, water and glycerin. What are the gage and absolute pressure at points 1 through 5 in pa. What are the equivalent columns of mercury (S.G = 13.6).



The standard atmosphere =101.3 Kpa.

$$P_1 = \underline{\underline{0 \quad gage}} = \underline{\underline{101.3\ KPa(abs)}}$$

$$P_2 = \gamma_g \times 2.5\ m = 1.25 \times 9.8 \frac{KN}{m^3} \times 2.5\ m$$

$$= \underline{\underline{30.625\ KPa}} = \underline{\underline{131.925\ KPa(abs)}}$$

$$P_3 = P_2 - \gamma_g \times 3 = 30.625 - (1.25 \times 9.81 \times 3)$$

$$= \underline{\underline{-6.125\ KPa}} = \underline{\underline{6.125\ KPa}} \quad (suction, vacuum) = \underline{\underline{95.175\ KPa(abs)}}$$

$$P_4 = P_3 \quad , \quad P_5 = P_4 - \gamma_w \times 2m = \underline{\underline{-25.745\ KPa}}$$

$$P_5 = \underline{\underline{75.555\ KPa(abs)}}$$